ESSFAR
ECOLE SUPERIEURE DES SCIENCES DE LA FINANCE, DE L'ASSURANCE ET DES RISQUES

Entrance exam in L3
September 2019 session

Mathematics test

Duration 4h

Note : In each exercise, questions numbered $1, 2, \ldots$ are independent

Exercice 1. *6 pts*

1. Let (u_n) be a sequence of positive terms. Show that the convergence $\sum_{n\in\mathbb{N}} u_n$ implies that of $\sum_{n\in\mathbb{N}} \frac{1}{n}\sqrt{u_n}$, and that the reciprocal is false.

2. Show that if the series $\sum_{n\in\mathbb{N}} v_n$ is alternating and the sequence $(|v_n|)$ is decreasing and tends to 0, then the series $\sum_{n\in\mathbb{N}}$ converges.

3. Represent the following domain
$$D = \{(x, y) \in \mathbb{R}^2;\ x \leq 1,\ y \leq 1,\ x + y \geq 1\}$$
and compute this integral
$$I = \int\int_D (x + y)dxdy$$
.

4. Let a, b and c be three reals. Determine the radius R and the convergence domain A of the power series $\sum_{n\in\mathcal{N}} a_n x^n$ where $a_n = e^{an^2+bn+c}$.

Exercice 2. *6 pts*

1. Let $v : \mathbb{R}^n \longrightarrow \mathbb{R}^n$ be a map. Connect the following props two by two.
   - A) v is continuous.
   - B) v admits continuous derivatives in all directions.
   - C) v is differentiable.
   - D) v is of class $\mathcal{C}^1$.

2. Consider the following set and matrix
$$V = \left\{ \begin{pmatrix} a & c \\ b & d \end{pmatrix},\ a,\ b,\ c,\ d \in \mathbb{R},\ a - d = 0 \right\}, \quad J = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$
and the map $g : V \times V \longrightarrow \mathbb{R}$, defined by $g(M, N) = trace(MJN)$.
   - (a) Show that g is bilinear. Is it symetric? Antisymetric?
   - (b) Determine a base B of V.
   - (c) Determine the matrix relative to the base B of the quadratic form q defined by
$$q(M) = g(M, M)$$
.
   - (d) Determine the kernel, rank and signature of q.

Exercice 3. 8 *pts*

1. Arrows are randomly shooted on a circular target of radius α, on which are drawn circles of radius $\frac{\alpha}{4}$, $\frac{\alpha}{2}$ and $\frac{3\alpha}{4}$. These circles delimit 4 different regions. It is assumed that the arrow reaches the target.

   Calculate the probability of reaching each region.

2. It is assumed that a newborn's weight is normally distributed from standard deviation 0.5 kg. The average weight of 49 newborns in March in a hospital was 3.6 kg.
   (a) Determine a 95% confidence interval of the average weight of a newborn in this hospital.
   (b) What would be the confidence level of a confidence interval of amplitude 0.1 kg centered at 3.6 kg of this average weight?

3. Consider the random vector (X, Y) having the following probability density

$$f(x, y) = 2e^{-x-y}1_{0<x<y}.$$

   (a) Determine marginal densities f_X and f_Y of X and Y respectively.
   (b) Determine the distribution functions F_X and F_Y of X and Y respectively.
   (c) Are X and Y independent?

4. Consider two random variables X uniformly distributed on $[0, 1]$, and Y taking values in $\{-1, +1\}$. Assume that

$$P(Y = +1|\ X = x) = \begin{cases} \frac{1}{4} & \text{si } \ x \in [0, \frac{1}{2}] \\ \frac{5}{6} & \text{si } \ x \in]\frac{1}{2}, 1]. \end{cases}$$

   Compute the following probability $P(X \in [a, b], Y = +1)$ as a function of a and b with $0 < a < b < 1$.